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Mooses, topology, and Higgs

Thomas Gregoire and Jay G. Wacker

Department of Physics, University of California
Berkeley, CA 94720, USA, and
Theory Group, Lawrence Berkeley National Laboratory
Berkeley, CA 94720, USA, and
Jefferson Physical Laboratory, Harvard University
Cambridge, MA 02138, USA
E-mail: gregoirt@socrates.berkeley.edu jgwacker@physics.berkeley.edu

Abstract: New theories of electroweak symmetry breaking have recently been constructed that stabilize the weak scale and do not rely upon supersymmetry. In these theories the Higgs boson is a weakly coupled pseudo-Goldstone boson. In this note we study the class of theories that can be described by theory spaces and show that the fundamental group of theory space describes all the relevant classical physics in the low energy theory. The relationship between the low energy physics and the topological properties of theory space allow a systematic method for constructing theory spaces that give any desired low energy particle content and potential. This provides us with tools for analyzing and constructing new theories of electroweak symmetry breaking.

Keywords: Spontaneous Symmetry Breaking, Beyond Standard Model, Higgs Physics, Chiral Lagrangians.

Contents

1. Introduction

The description of electroweak symmetry breaking in the Standard Model, in terms of a fundamental scalar Higgs field, suffers from a stability crisis. The quadratically divergent radiative corrections to the Higgs mass suggest that the description of TeV scale physics in the Standard Model is incomplete. New physics at the TeV scale must emerge to stabilize the weak scale. Recently, a qualitatively new category of realistic theories of electroweak symmetry breaking has been introduced [1]. These models, based on deconstruction [2, 3, 4] and the physics of "theory space" [5]–[12] offer a new mechanism for softening the quadratic divergences in the Higgs mass. Electroweak symmetry breaking is accomplished with naturally light Higgs bosons that descend from non-linear sigma model fields whose mass is protected by "chiral" symmetries of the sigma model. The first attempts at models of this kind were the "composite Higgs" theories [14, 15, 16] that required fine tuning to keep the Higgs light. More recently, models similar in spirit to the theory space models and using the same group theory structure as the composite Higgs model have been developed [17]. In all of these theories, the physics is perturbative at energies parametrically above the TeV scale, ultimately requiring an ultraviolet completion near $\sim 10\,\mathrm{TeV}$ where the non-linear sigma model fields become strongly coupled. However, the physics of electroweak symmetry breaking and the new physics at the TeV scale are weakly coupled and do not depend on the ultraviolet completion. These models are fully realistic, incorporating fermion masses without producing dangerous flavour-changing neutral currents in the low energy theory.

JHEP08(2002)019

The general structure of these models is characterized by a "theory space", consisting of sites, lines and faces. Each site represents a gauge group, each line represents a non-linear sigma model link field transforming under the gauge groups at the ends of the line, and each face corresponds to "plaquette" operators involving a trace of products of the link fields bounding the face. The little Higgs descend from the link fields, while their quartic coupling arise from the plaquette interactions. Based on deconstructing extra dimensional intuitions, the models used in [1, 19] were $N \times N$ deconstructed torus. The basic ingredients that make this class of models successful theory of electroweak symmetry breaking are the absence of one loop quadratic divergences in the Higgs mass, guaranteed by the approximate chiral symmetries, and the presence of large quartic self interaction for the Higgs.

In this paper, we seek a way of extracting the low energy physics of general theory spaces in order to decide which spaces can be used for electroweak symmetry breaking. We also develop a method for building theory spaces with a given low energy particle content and potential. The spectrum of electroweak symmetry breaking theories based on theory space is characterized by two or more Higgs doublets at roughly 100 GeV and at least one TeV scale particle for each quadratic divergence of the low energy theory. In contrast with supersymmetric theories, quadratic divergences are canceled by 'partners' of the same spin.

In section 2, we review the structure of theory space and present a systematic procedure to calculate the moduli space of general theory space, allowing us to obtain the low energy potential of the theory. We illustrate this procedure with several examples. We then reverse the logic and show how to build theory spaces that possess arbitrary low energy physics.

In section 3, we analyze the structure of radiative corrections in little Higgs model, and present two simple rules that ensure that a theory space is free of quadratic divergences at one loop. In section 4 we discuss how to include Yukawa couplings so that they do not reintroduce one loop quadratic divergences. We also show that it is possible for fermions to generate the plaquette potential.

Finally, in section 5 we discuss how to lift unnecessary states out of the low energy theory and into the 1 TeV range. When the gauge symmetry is reduced at one site new plaquette potentials are allowed that can differentiate between the adjoint states and Higgs (these are the $\mathbf{T_8}$ plaquettes of [1, 19]). This allows us to build models free of light triplet and singlet scalars that were present in other little Higgs models constructed from theory space [1, 19, 20]. In particular we present an extension of the two sites model of [20] where the $\sim$ 100 GeV triplet and singlet scalars of [20] are pushed to the TeV scale.

2. Topology and theory space

There are general statements we can make about the existence of little Higgs and their potentials from the structure of theory space alone. Understanding the general structure of theory space and its relation to the low energy dynamics will allow us to classify the little Higgs theories and determine if they are viable models of electroweak symmetry breaking.

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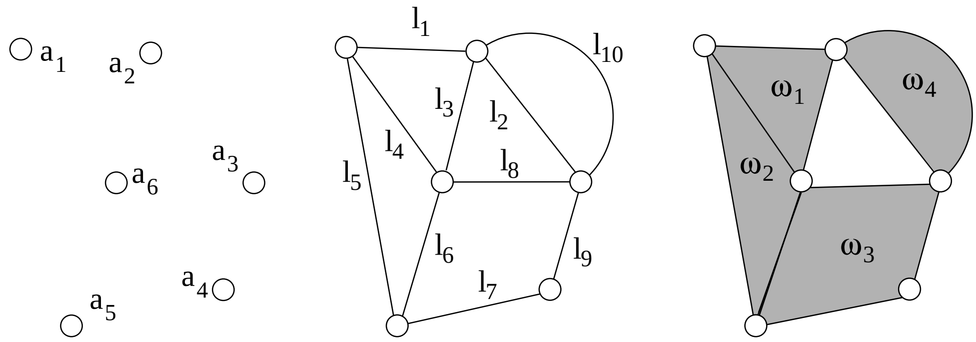

Figure 1: The geometry of theory space being built up from points, lines, and faces. These geometrical objects are identified as gauge groups, fields, potentials in the action.

The physics of little Higgs models is specified by the gauge structure, the link variables and the scalar potential, these define theory space by points, lines, and faces, respectively. Gauge groups are labeled by points: $G_{\mathbf{a}}$. Link variables are labeled by line segments, $\Sigma_{\mathbf{l}} = \exp(i\pi_{\mathbf{l}})$ that transform as bifundamentals under the endpoints of the line $l = (\mathbf{a}, \mathbf{b})$

$$\Sigma_l \to g_{\mathbf{a}} \Sigma_{\mathbf{l}} g_{\mathbf{b}}^\dagger . \tag{2.1}$$

Finally the plaquette potentials are interpreted as shaded in faces and are the product of the link fields that bound the faces: $W_\omega = \Sigma_{\mathbf{l_1}} \dots \Sigma_{\mathbf{l_N}}$. The lagrangian for a theory space is given by:

$$\mathcal{L} = \sum_{\mathbf{a}} \frac{-1}{2g_{\mathbf{a}}^2} \text{ Tr } F_{\mathbf{a}}^2 + \sum_{\mathbf{l}} \frac{f_{\mathbf{l}}^2}{4} \text{ Tr } \left|D^\mu \Sigma_{\mathbf{l}}\right|^2 + \sum_{\omega} \lambda_\omega f^4 \text{ Tr } W_\omega + \text{ h.c.} \tag{2.2}$$

The full gauge group of a theory space is given by the product of the gauge groups associated with each sites: $G_{\text{total}} = \prod_{\mathbf{a}} G_{\mathbf{a}}$. However, only a small subgroup of this gauge symmetry is realized linearly on the π_l. This is the low energy unbroken subgroup under which:

$$\Sigma_l \to g \Sigma_l g^\dagger \tag{2.3}$$

So long as all the link fields connect two sites, for each disconnected component of theory space there is an unbroken gauge symmetry corresponding to the diagonal subgroup of the product of all the gauge groups associated with the sites in the given component.

To build realistic models of electroweak symmetry breaking, the Higgs must transform as $\mathbf{2_{1/2}}$ under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. However, if all the gauge groups of a theory space are the same and the link fields transform as bifundamentals, the scalars of the theory will be adjoints under the unbroken gauge group. One way of solving this problem is to reduce the gauge symmetry at one of the sites. We will in general take all the sites to be SU(3) gauge group except one where we will gauge only $\mathrm{SU}(2) \times \mathrm{U}(1)$. The link fields are 3×3 matrices and a link that touch the site of reduced gauge symmetry transform as:

$$\Sigma_l \to h_{\mathrm{SU}(2)} e^{\frac{i}{6}\theta \mathbf{T_8}} \Sigma_l g_{\mathrm{SU}(3)}^\dagger \tag{2.4}$$

where $\mathbf{T_8} =$ diag $(1,1,-2)$[1] and where $h_{\mathrm{SU}(2)}$ commutes with $\mathbf{T_8}$. The unbroken diagonal subgroup is the electroweak $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ and the scalars of the theory will decompose into triplets, doublets and singlet of the unbroken SU(2). The site of reduced symmetry allows for interesting possibilities that will be discuss in section 5, but for the discussion of the present section, it is irrelevant.

We want to study the low energy physics of these models at scales beneath the modes that have tree-level masses. This can be done by integrating out the massive modes, but this is a cumbersome procedure. To integrate out the heavy modes and have the low energy theory, it is necessary to find the full spectrum of the theory and to find all trilinear interactions involving two light scalars and a heavy scalar and all quartic interactions with only light scalars. When heavy scalars are integrated out, trilinear interactions involving two light scalars and a heavy scalar can exactly cancel a quartic interaction with only light scalars. Verifying which light scalars have a tree-level quartic interaction is therefore rather intricate and avoiding this procedure is desirable. The moduli space captures much of the relevant low energy physics in the scalar sector and calculating this space will be the primary goal of this section. We first explain the procedure for calculating the moduli space of a general theory space and then illustrate it with several examples.

The moduli space is gauge invariant, meaning that we can gauge fix in any convenient manner. If theory space is arc-wise connected, then it is possible to draw a simply connected line through theory space that touches every point only once. All the links along this line can be gauged away and this procedure completely fixes the gauge. When theory space is not arc-wise connected, there is no simple rule and we must gauge fix by hand. To find the physical spectrum it is more convenient to go to unitary gauge which is a more difficult task.

After gauge fixing, we minimize the plaquette potential by setting the products of link fields corresponding to faces to the identity matrix. This minimization will fix most of the link fields. The interesting part of the moduli space is then specified by relations between the remaining link fields. The flat directions of this moduli space are the little Higgs of the theory. To reproduce this moduli space in the low energy effective action, we include the relations as a potential so that as we go off the moduli space there is an energy cost. Theories that have no relations must have potentials generated radiatively and therefore have the same generic problems that typical pseudo-Goldstone bosons suffer from — that it is not possible to have a parametric separation between the cut-off and the vacuum expectation value. Identifying interesting little Higgs theories reduces to finding theory spaces with interesting relations.

The procedure of gauge fixing then minimizing the potential is precisely equivalent to calculating the fundamental group of theory space (or first homotopy group), see chapter four of [24] for more details. In the equivalence, little Higgs are non-contractible cycles on theory space and the low energy potential is the relation in the homotopy group. This links all the relevant low energy physics to topological properties and is independent of the tiling of theory space chosen. When the tilings are taken to be large, the physics of

[1]The normalization of the U(1) is to have the Higgs doublet have hypercharge 1/2.

theory space is identical to the physics of an extra dimension. In the extra dimensional picture, the little Higgs are flat gauge connections and are classified by the fundamental group. In the extra dimensional limit the physics of theory space and of extra dimensions are identical, however, this equivalence is valid for any theory space, including ones that bear no resemblance to an extra dimension. The relation between the low energy physics and the fundamental group provides a practical way for both analyzing models as well as constructing new models.

Circles and Disks

A theory space that is topologically a circle is an example of a theory with a little Higgs. This theory was analyzed in [1] and in more depth in [23]. The link fields transform as $\Sigma_a \sim \overline{\Box}_a \times \Box_{a+1}$ and can be written as exponential: $\Sigma_a = \exp i\pi_a$. The lagrangian is given by:

$$\mathcal{L}_{S^1} = \sum_a -\frac{1}{2g_a^2}\ \mathrm{Tr}\ F_a^2 + \sum_a \frac{f_a^2}{4}\ \mathrm{Tr}\ D^\mu \Sigma_a D_\mu \Sigma_a^\dagger + \cdots \tag{2.5}$$

The ellipses represent higher dimension operators that are irrelevant at low energies. The residual gauge symmetry indicates that there is a massless gauge boson and $N-1$ massive vector bosons. Of the N non-linear sigma model fields, $N-1$ are eaten by the massive vector bosons and one physical massless scalar is left over. Furthermore, from eq. 2.5, we see that this scalar does not have a tree-level potential because there are no plaquettes.

We will choose to gauge fix in a manner that eliminates as many of the link fields as possible. Starting with Σ_1, we can choose gauge transformations g_1 and g_2 so that $\Sigma_1 = \mathbb{1}$. Similarly it is possible to gauge away Σ_2 with g_3. It is possible to gauge away all but one of the links. It is not possible to gauge away the last field because the last link closes the circle and the gauge freedom for g_1 had already been used to fix Σ_1. In this gauge the physical scalar, $\Sigma = \exp(i\sigma)$, mixes with the gauge fields, therefore this gauge is inconvenient for calculating the physical spectrum of gauge bosons. Unitary gauge is more convenient for computing the spectrum because there is no vector-scalar mixing. We can interpret Σ as a classical modulus of the theory. This classically massless mode is a pseudo-Goldstone boson called a little Higgs. The low energy effective action is just:

$$\mathcal{L}_{\mathrm{LE}} = -\frac{1}{2g_D^2}\ \mathrm{Tr}\ F^2 + \frac{f_{\mathrm{LE}}^2}{4}\ \mathrm{Tr}\ \left|D_\mu \Sigma\right|^2 + \cdots \tag{2.6}$$

where σ is an adjoint under the unbroken gauge symmetry. A potential for σ that lift the moduli space will be generated at one loop, however, the only gauge invariant operators are of the form $\mathrm{Tr}\ \Sigma \sim \cos(\sigma)$. The pseudo-Goldstone boson, σ, can not have significant self-interaction without having a significant mass. This form of the low energy potential is too constrained to be used for electroweak symmetry breaking as it does not allow for a parametric separation between the vacuum expectation value of the little Higgs and the cutoff of the theory.

Next, consider a theory space with the topology of a disk by adding the plaquette $\mathrm{Tr}\ \Sigma_1 \ldots \Sigma_N$. This space has no non-contractible cycles and therefore has no little Higgs. After filling in theory space with more sites, links and plaquettes, we can make "holes" in a

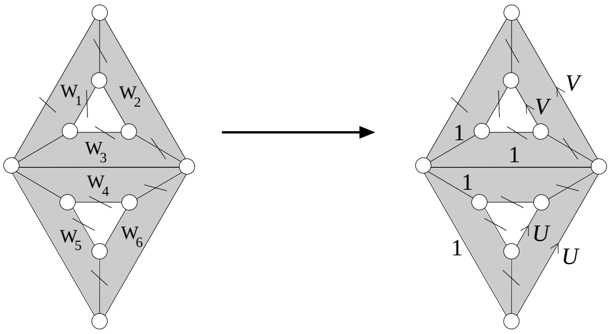

Figure 2: A deconstruction of a disk with two holes.

disk by omitting plaquettes. This creates non-contractible cycles in theory space. A theory space with the topology of a disk with two holes is shown in figure 2. We can gauge fix by drawing a line through theory space that goes through every points. Upon minimizing the potential, there are two moduli corresponding to the two non-contractible cycles. These moduli are arbitrary non-linear sigma model fields because there is no relation for the homotopy group. This means that there is no tree-level potential in the low energy theory and any deconstruction of this space will be unsuitable for electroweak symmetry breaking. The existence of two little Higgs does not guarantee a tree level potential. Because of the homotopy arguments, a disk with h holes will have h little Higgs, but none of these scalars will ever have a tree-level potential because the fundamental group of theory space is (in the notation of [24]) $\pi_1 = \{C_1, \dots, C_h : -\}$, where C_i are the non-contractible cycles on theory space and "$-$" represents that there is no relation between the cycles.

Torus

A theory space that is topologically a torus has two little Higgs. The primary new feature with this theory space is the appearance of a relation in the definition of the fundamental group:

$$\pi_1(T^2) = \{U, V : UVU^{-1}V^{-1}\}\,. \tag{2.7}$$

This will lead to a tree-level potential for the little Higgs associated with the cycles U and V. Consider an $N \times N$ sites deconstruction of a torus with the sites labeled (a, b). We will take our fields to be $U_{(a,b)} \sim \overline{\Box}_{(a,b)} \times \Box_{(a+1,b)}$ and $V_{(a,b)} \sim \overline{\Box}_{(a,b)} \times \Box_{(a,b+1)}$. To make this space topologically a torus, we periodically identify $(a, b) \equiv (a + N, b) \equiv (a, b + N)$. This theory breaks the G^{N^2} gauge symmetry down to the diagonal subgroup G_D. There are $N^2 - 1$ Nambu-Goldstone bosons that are eaten by the massive vectors. From the continuum limit, we suspect that the potential gives mass to $N^2 - 1$ of the physical modes leaving two modes massless. The lagrangian for theory space is given by:

$$\begin{aligned}\mathcal{L}_{T^2} = {} & \sum_{a,b} -\frac{1}{g^2_{(a,b)}} \text{ Tr } F^2_{(a,b)} + \sum_{a,b} \frac{f^2_{U(a,b)}}{4} \text{ Tr } \left|D_\mu U_{(a,b)}\right|^2 + \frac{f^2_{V(a,b)}}{4} \text{ Tr } \left|D_\mu V_{(a,b)}\right|^2 + \\ & + \sum_{a,b} \lambda_{(a,b)} f^4 \text{ Tr } W_{(a,b)} + \text{ h.c.}\end{aligned} \tag{2.8}$$

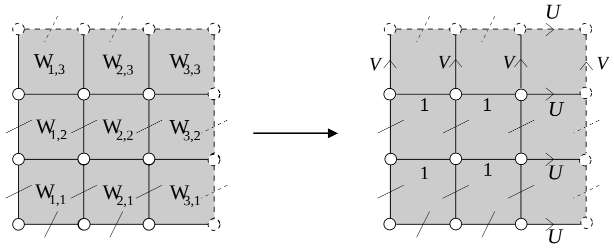

Figure 3: Gauge fixing of the torus where crossed lines are gauged to the identity. The plaquettes are then minimized. Plaquette $W_{3,3}$ forces $UVU^{-1}V^{-1} = 1$.

where

$$W_{(a,b)} = U_{(a,b)}V_{(a+1,b)}U^{\dagger}_{(a,b+1)}V^{\dagger}_{(a,b)}\,. \tag{2.9}$$

We can see that there are two massless modes in this theory from the fact that the N^2 plaquettes terms $W_{(a,b)}$ give masses to $N^2 - 1$ of the scalars.

To analyze the model in more details, we first gauge fix to eliminate as many fields as possible. We then minimize the potentials by requiring that $W_{(a,b)} = \mathbb{1}$. This procedure is illustrated in figure 3. We find that the vacuum is given by:

$$UVU^{-1}V^{-1} = \mathbb{1}\,. \tag{2.10}$$

This is the classical moduli space: two unitary matrices that commute. To enforce this in the low energy effective action we include this relation as a potential so that there is an energy cost for going off the moduli space:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2g_D^2}\ \mathrm{Tr}\ F^2 + \frac{f_U^2}{4}\ \mathrm{Tr}\ \left|D_\mu U\right|^2 + \frac{f_V^2}{4}\ \mathrm{Tr}\ \left|D_\mu V\right|^2 + \lambda_{\text{eff}} f^4\ \mathrm{Tr}\ UVU^{\dagger}V^{\dagger} +\ \text{h.c.} \tag{2.11}$$

There is now a tree-level quartic potential, and masses are induced radiatively. This allows a hierarchy between the cut-off and the vacuum expectation value of little Higgs that will allow stabilization of the electroweak scale.

If one of the plaquette couplings of the torus is taken to vanish, the topology of theory space has changed. In figure 4 we compute the fundamental group and find that there is no relation between the cycles and therefore no low energy potential for the little Higgs. We can calculate the coefficient of the potential for a general torus through a linearized analysis by diagonalizing the scalar mass matrix and then integrating out the massive modes. We find that the coefficient of the potential λ_{eff} is given by:

$$\lambda_{\text{eff}}^{-1} = \sum_{(a,b)} \lambda_{(a,b)}^{-1}\,. \tag{2.12}$$

We see that if any coefficient vanishes, then the low energy potential vanishes precisely matching the topological argument.

Toroidal theory spaces of the type shown in figure 3 are not the simplest theory space having the fundamental group of the torus (eq. 2.7). Consider a theory space with two sites, four bi-fundamental links X_i and two plaquettes:

$$V(X) = -\lambda_1 f^4\ \mathrm{Tr}\ X_1X_2^{\dagger}X_3X_4^{\dagger} - \lambda_2 f^4\ \mathrm{Tr}\ X_2X_3^{\dagger}X_4X_1^{\dagger}\,. \tag{2.13}$$

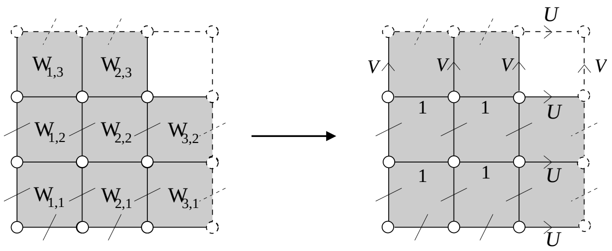

Figure 4: Gauge fixing of the torus where crossed lines are gauged to the identity. The plaquettes are then minimized. Since plaquette $W_{3,3}$ is absent, there is no relation and the moduli space is arbitrary U and V and there is no low energy potential for the little Higgs.

This theory was first analyzed in [20]. It can be easily analyzed by first gauge fixing X_1 to $\mathbb{1}$ and then solving for $X_4 = X_2^\dagger X_3$. We are left with the relation

$$X_2 X_3^\dagger X_2^\dagger X_3 = \mathbb{1} \tag{2.14}$$

which is the commutator potential of a torus. One can show that this theory space is related to the 2×2 torus by orbifolding by a translational symmetry that sends all points $(i, j) \to (i + 1, j + 1)$. This symmetry acts freely and does not change the homotopy of the space and therefore does not change the little Higgs or their self-interaction. The physics of this theory space is studied in more details in [20].

2.1 Reverse engineering

Finding the low energy physics from a theory space is a straight-forward procedure of gauge fixing then minimizing the potential. There is also an intuitive procedure for taking a low energy potential in the form of a product of nonlinear sigma model fields and finding a high energy theory that produces it at low energy. This construction is reverse engineering the theory space from the low energy potential. The most interesting theories to consider are the minimal ones. It is not difficult to conclude that the simplest potential that is viable for electroweak symmetry breaking is $\text{Tr}\ UVU^\dagger V^\dagger$. This means that the theory space that produces this potential is homotopically equivalent to the torus. The simplest such theory with more than one site is the two sites four links model of the previous section. To illustrate this construction we will use non-minimal models that are still viable models of electroweak symmetry breaking.

Given a set of non-linear sigma model light fields X_i and a potential $V(X_i)$ that is a product of the fields and their inverses, we draw out the potential as a polygon with each side being the corresponding link field. Each link begins and ends at the same site, a. For instance consider three light fields X, Y, Z and a potential $V = \text{Tr}\ XYZX^{-1}Y^{-1}Z^{-1}$. In figure 5, we draw out the unfolded and folded versions of this theory space.

Any theory space that tiles this minimal version of theory space will have the same low energy potential. Dividing the plaquettes and links by placing new points and links in theory space will not change the low energy potential. For instance we can divide the theory space in figure 5 up in figure 6. We can also build different theory spaces that have

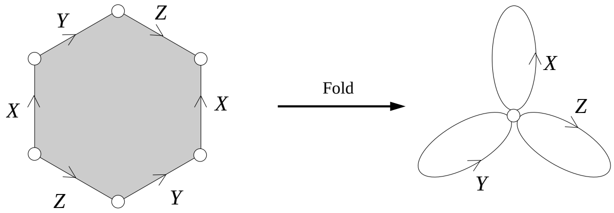

Figure 5: The minimal theory space with three cycles, X, Y, Z and the potential $V =$ Tr $XYZX^{-1}Y^{-1}Z^{-1}$. The arrows along the links indicate whether the fields are X_i or X_i^{-1}.

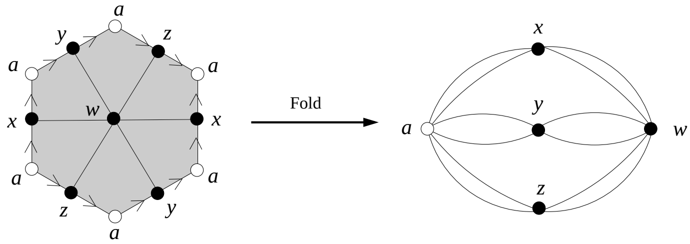

Figure 6: A larger deconstruction of the $XYZX^{-1}Y^{-1}Z^{-1}$ three cycle model where we have introduced four new gauge groups (x, y, z, w). The plaquette structure in the unfolded deconstruction is obvious, but the field content is harder to visualize because of the identifications. In the folded version, the gauge and field content is clear, but the plaquettes structure is obscured.

the same low energy physics as the torus. Figure 7 shows three such theory spaces. They are obtained by requiring a low energy potential of the form $XYX^{-1}Y^{-1}$ and tiling the original construction in different manners.

Finally, some spaces have fundamental groups with more than one relation. To construct theory spaces that are homotopically equivalent to these spaces we draw the multiple relations as disjoint diagrams although theory space is connected. In figure 8 a theory space with a fundamental group

$$\pi_1 = \{X, Y, Z : XYX^{-1}Y^{-1}, XZX^{-1}Z^{-1}\} \tag{2.15}$$

is constructed.

We have shown how to analyze and build theory spaces with classically massless Higgs and order one quartic interactions. This is not sufficient to ensure that a theory space can be used for electroweak symmetry breaking, as radiative corrections might make the Higgs too heavy. We will show in the next section that in order for that not to be the case, theory spaces must satisfy mild constraints but there is still an arbitrariness to the theory spaces that produce a given low energy physics.

3. Radiative corrections

Without gauge couplings and plaquette interactions, a theory space with M link fields has a G^{2M} global chiral symmetry, under which each link field transform as bifundamental under independent global symmetries:

$$\Sigma_l \to L_l \Sigma_l R_l^\dagger \tag{3.1}$$

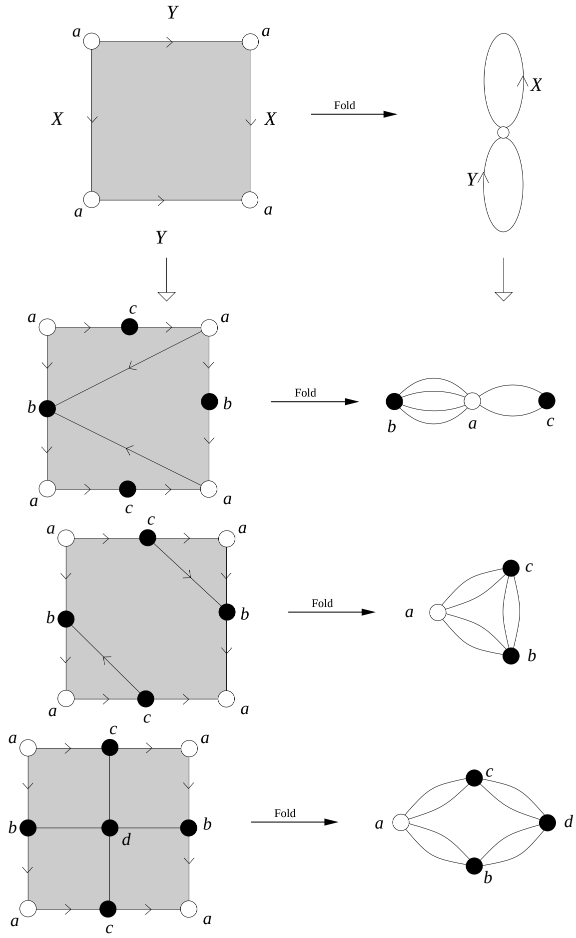

Figure 7: Alternative deconstructions of a torus. The last figure is the 2×2 torus of [19].

Without couplings, the link fields are exact Goldstone bosons with only derivative interactions. Once gauge and plaquette couplings are included, some set of the chiral symmetries are broken. The coupling constants may be viewed as spurions that give rise to masses and non-derivative interactions. The essential feature of little Higgs theories that guarantees ultraviolet insensitivity is that generation of operators containing mass terms for the little Higgs requires many spurions. Consequently, since ultraviolet physics is analytic in the parameters, quadratically divergent contributions to the little Higgs mass are suppressed by many loop factors.

When building theory spaces there must be enough spurions so that there are no one loop quadratic divergences. However, even if the one loop quadratic divergences are absent, generically there will be a one loop finite contribution to the little Higgs mass so long as the little Higgs is not an exact Goldstone boson. Infrared physics is not analytic in the parameters and the finite contribution is of the order of:

$$m_{\rm LH}^2 \sim \frac{g^2}{(4\pi)^2} M_{\rm H}^2 \tag{3.2}$$

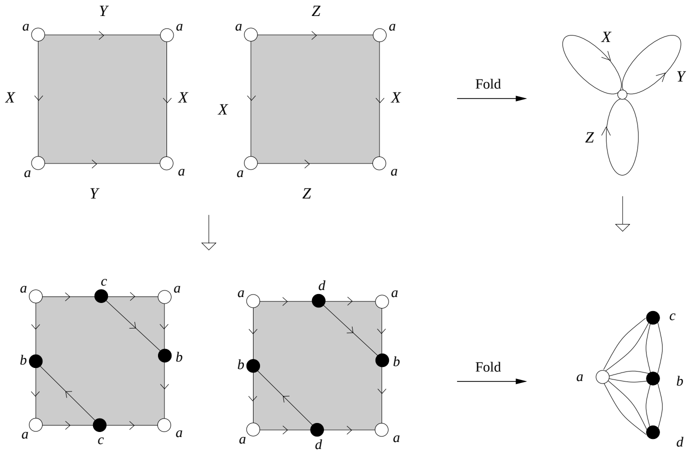

Figure 8: Construction of a theory space with relations $XYX^{-1}Y^{-1}$ *and* $XZX^{-1}Z^{-1}$. Note that the two squares must be tiled with different links.

where M_H is the mass of lightest new state which generically is of order $M_H \sim gf$. Using this relation we find:

$$m_{\mathrm{LH}}^2 \sim \frac{g^2}{(4\pi)^2} g^2 f^2 \sim \frac{g^4}{(4\pi)^4} \Lambda^2 \tag{3.3}$$

where $\Lambda \sim 4\pi f$ is the ultraviolet cut-off of the theory. The infrared contributions are of the same order of magnitude as a two loop quadratic divergence. Therefore, it is unnecessary to eliminate anything but the one loop quadratic divergence. The only benefit of eliminating divergent contribution of higher loop order would be that the little Higgs mass would be calculable because the mass would be dominated by infrared physics, as opposed to having ultraviolet and infrared physics providing parametrically the same contribution. Another possible reason for eliminating more than one loop quadratic divergences would be if a coupling was so strong so that loops involving this coupling were not suppressed.

Having to only eliminate the one loop quadratic divergences, the constraints on theory space are very mild and can be stated simply:

Gauge Sector: Every link must connect two different sites.

Scalar Sector: No plaquette can contain the same link twice.

We can prove these rules by computing the quadratically divergent part of the one loop Coleman-Weinberg potential. We turn on a little Higgs background fields and calculate $\mathrm{Tr}\ M^\dagger M$ where M is the mass matrix of the theory in the presence of the background.

We first consider the gauge sector and show that gauge interactions never produce one loop quadratic divergences so long as all the link connect two different sites or equivalently all link fields are in bifundamentals as opposed to adjoint representations. Consider a link field between two different sites i and j. The gauge boson mass matrix comes from the covariant derivative, $A_i^a M^{2\,ij}_{\ ab}[\tilde{U}] A_j^b$, where a, b are gauge indices and

$$M_{ab}^2[\tilde{U}] = \frac{f^2}{4} \begin{pmatrix} \frac{1}{2} g_i^2 \delta_{ab} & -g_i g_j m_{ab}[\tilde{U}] \\ -g_i g_j m_{ab}^\dagger[\tilde{U}] & \frac{1}{2} g_j^2 \delta_{ab} \end{pmatrix} \qquad m_{ab}[\tilde{U}] = \ \mathrm{Tr}\ \mathbf{T}_a \tilde{U} \mathbf{T}_b \tilde{U}^\dagger$$

The important point is that diag M^2 is always independent of the background field, $\tilde{U}$, and therefore will never produce a one loop quadratic divergence for any link field mass. If a field is in the adjoint, then this argument will break down and a one loop quadratic divergence will appear.

We now turn to the scalar sector. Consider an arbitrary plaquette:

$$V(U_i) = -\lambda f^4 \text{ Tr } M_1 U_1 \dots M_N U_N + \text{ h.c.} \tag{3.4}$$

where M_i are arbitrary matrices. We rewrite the link fields as a linearized fluctuations, u_i, and a background fields, $\tilde{U}_i$: $U_i = \exp(iu_i)\tilde{U}_i$. By dividing U_i in this way, the background field drops off the kinetic term and we can extract the mass of u_i directly from the potential without having to worry about putting the kinetic term in canonical form. We expand out the plaquette to quadratic order in the fluctuations and find the mass matrix, $u_i^a {M^2}_{ab}^{ij} u_j^b$. The diagonal of the mass matrix is

$$\text{diag } {M^2}_{ab}^{ij} \sim \lambda f^2 \text{ Tr } M_1\tilde{U}_1 \dots M_i \mathbf{T}_a \mathbf{T}_a \tilde{U}_i M_{i+1}\tilde{U}_{i+1} \dots M_N \tilde{U}_N \tag{3.5}$$

where $\mathbf{T}_a$ are gauge group generators. Then summing over the diagonal entries of the mass matrix and using $\sum_a \mathbf{T}_a^2 \sim \mathbb{1}$, we find Tr $M^2 \propto M_1\tilde{U}_1 \dots M_i\tilde{U}_i M_{i+1}\tilde{U}_{i+1} \dots M_N\tilde{U}_N$ which is just the plaquette operator. Since, by definition, the plaquettes do no contain mass term for the little Higgs, this shows that plaquettes never produce one loop quadratic divergences to the little Higgs mass unless fields appear in plaquettes more than once. If a field appears in a plaquette more than once, than this argument will break down because the mass matrix will have a more complicated form with $\tilde{U}$ dependent diagonal entries.

We are left with two requirements for a theory space to have no one loop quadratic divergences: that no link begins and ends on the same point — that no link field is in an adjoint representation, and that no plaquette contains a link twice. These constraints can be easily satisfied, even with small theory spaces. These requirements place restrictions on the minimal field content at the TeV scale. For instance, there must be at least a second $\mathrm{SU}(2) \times \mathrm{U}(1)$ (or $\mathrm{SU}(3)$) gauge symmetry broken around the TeV scale with massive gauge bosons W' and B'. There must also be a massive multiplet of triplet, doublet and singlet scalars ϕ, h, and η at the TeV scale to ensure that the scalar potential does not induce a one loop quadratic divergence.

4. Fermions

The Standard Model Higgs is a pseudo Goldstone boson in little Higgs models and has the same quantum numbers as the kaon. The Higgs mass is only protected from one loop quadratic divergences if we preserve some of the global SU(3) chiral symmetry. In the gauge and scalar sectors of these theories this was automatic at one loop, however, in the fermion sector one loop quadratic divergences are possible if all the SU(3) chiral symmetry that is protecting the Higgs mass is broken by one coupling.

It is useful to write the Standard Model fermions as incomplete SU(3) triplets at the $\mathrm{SU}(2) \times \mathrm{U}(1)$ site $\mathbf{0}$ in order to make manifest the SU(3) symmetries we want to preserve in the Yukawa couplings.

$$Q = \begin{pmatrix} q \\ 0 \end{pmatrix} \qquad U^c = \begin{pmatrix} 0 \\ 0 \\ u^c \end{pmatrix} \qquad D^c = \begin{pmatrix} 0 \\ 0 \\ d^c \end{pmatrix} \qquad L = \begin{pmatrix} l \\ 0 \end{pmatrix} \qquad E^c = \begin{pmatrix} 0 \\ 0 \\ e^c \end{pmatrix} \tag{4.1}$$

Under a $\mathrm{U}(1)_{\mathbf{0}}$ transformation, θ, these fields transform as:

$$Q \to e^{\frac{i}{6}\theta} Q \qquad U^c \to e^{-\frac{2i}{3}\theta} U^c \qquad D^c \to e^{\frac{i}{3}\theta} D^c \qquad L \to e^{-\frac{i}{2}\theta} L \qquad E^c \to e^{i\theta} E^c \tag{4.2}$$

At low energies, the effective coupling to the little Higgs is just through a Wilson line operator W that stretches from site $\mathbf{0}$ back to site $\mathbf{0}$. Under a $\mathrm{U}(1)_{\mathbf{0}}$ transformation, W transforms as:

$$W \to \exp\left(\frac{i}{6}\mathbf{T_8}\theta\right) \; W \; \exp\left(-\frac{i}{6}\mathbf{T_8}\theta\right) \tag{4.3}$$

with $\mathbf{T_8} = \mathrm{diag}\,(1,1,-2)$. Let us introduce projections matrices $P_1 = \mathrm{diag}\,(1,1,0)$ and $P_2 = \mathrm{diag}\,(0,0,1)$. The gauge invariant Yukawa couplings[2] are given by:

$$y_u f Q^T P_1 W P_2 U^c \qquad y_d f Q^T P_1 W^* P_2 D^c \qquad y_e f L^T P_1 W^* P_2 E^c\,. \tag{4.4}$$

These couplings arise from an ultraviolet completion in the $\sim 10\,\mathrm{TeV}$ range. Having particles that carry flavour at this scale can produce unacceptably large flavour changing neutral currents [18]. Flavour physics places constraints on possible completions. A simple solution is to complete these theories into supersymmetric linear sigma models at this scale. These couplings introduce quadratic divergences of the form:

$$\mathcal{L}_{\mathrm{eff}} = \frac{y_f^2}{16\pi^2} f^2 \Lambda^2 \;\mathrm{Tr}\; P_1 W P_2 W^\dagger\,. \tag{4.5}$$

This is just the usual quadratic divergence to the Higgs mass coming from Yukawa couplings. For everything, but the top quark, the Yukawa couplings are small enough so that the quadratic divergences are small enough to be ignored. For the top quark, removing the one-loop quadratic divergence is of paramount importance. A solution was discussed in [1, 19] where additional Dirac fermions were introduced on intermediate SU(3) sites. The key ingredient was preserving at least one of the SU(3) global symmetries protecting the Higgs mass. In this note we will consider an alternative mechanism. We can imagine introducing an Dirac SU(2) doublet S, S^c such that we complete U^c into an SU(3) triplet:

$$U^c = \begin{pmatrix} S^c \\ u^c \end{pmatrix}. \tag{4.6}$$

With the lagrangian:

$$\mathcal{L}_{\mathrm{top}} = y_u f Q^T P_1 W U^c + m_S S S^c + \text{ h.c.} \tag{4.7}$$

[2] The low energy $llhh$ dimension five Yukawa coupling that gives a neutrino mass is written in this language as $L^T P_1 W^T P_2 W P_1 L$.

the one-loop quadratic divergence is $\mathrm{Tr}\ P_1 W W^\dagger$, where there is not a second projection matrix because of the global SU(3) symmetry of U^c. If W is unitary, i.e. a product of link fields, then this removes the one loop quadratic divergence. The chiral symmetry protects the little Higgs' mass. Similarly to the gauge and scalar sectors, we now have a rule for avoiding quadratic divergence in the fermionic sector:

Fermion Sector: The top Yukawa couplings must preserve either a left ($W \to LW$) or right ($W \to WR^\dagger$) chiral symmetry.

The effective top quark Yukawa coupling is

$$y_{\mathrm{eff}}^{-2} = (y_u)^{-2} + \left(\frac{m_S}{f}\right)^{-2} \tag{4.8}$$

meaning that m_S/f and y_u should both be at least order unity to have an adequately large top Yukawa coupling.

4.1 Plaquettes from Yukawa interactions

We now restrict ourselves to the model of [20] involving two sites and four links. If we consider an alternate Wilson line: $W_1 = X_1 X_2^\dagger + X_4 X_3^\dagger$, for the top quark then we find a quadratic divergence in eq. 4.7. However, this divergence is to the operator $\mathrm{Tr}\ P_1 X_1 X_2^\dagger X_3 X_4^\dagger$ – one of the requisite plaquettes. This indicates that with this choice of Yukawa coupling, it is *unnatural* for the coefficient of this plaquette to be *small.* In other words, if we choose to set the tree-level coefficient of the operator to zero, it will be generated at one-loop with an order f^4 coefficient, precisely the value we want. This is only one of the plaquettes in the model of [20], but with a slightly more elaborate fermion sector it is possible to generate both plaquettes from the top sector alone. The emphasis is that plaquette operators are naturally generated with a sizeable coefficients from physics below 10 TeV.

A simple realization of top physics inducing the entire Higgs potential uses an additional colored weak doublet Dirac fermion $\tilde{q}$, $\tilde{q}^c$. Introducing two Wilson lines $W_1 = c_1 X_1 X_2^\dagger + c_1' X_4 X_3^\dagger$ and $W_2 = c_2 X_4 X_1^\dagger + c_2' X_3 X_2^\dagger$, we couple one Wilson line to each two quark doublet in the Yukawa interactions:

$$\mathcal{L}_{\mathrm{top}} = y_u f Q W_1 U^c + \tilde{y}_u f \tilde{Q} W_2 M U^c + m_{\tilde{q}} \tilde{q} \tilde{q}^c + m_S S S^c \,. \tag{4.9}$$

where $M = \mathrm{diag}\ (1,1,i)$ is a unitary matrix of phases and $\tilde{Q} = (\tilde{q}, 0)$. The one loop quadratic divergence gives each plaquette:

$$\mathcal{L}_{\mathrm{eff}} = \frac{y_u^2}{16\pi^2} f^2 \Lambda^2\ \mathrm{Tr}\ P_1 |W_1|^2 + \frac{\tilde{y}_u^2}{16\pi^2} f^2 \Lambda^2\ \mathrm{Tr}\ P_1 |W_2|^2 \,. \tag{4.10}$$

The one loop Coleman-Weinberg analysis gives a negative contribution to the Higgs mass driving it negative and breaking electroweak symmetry.

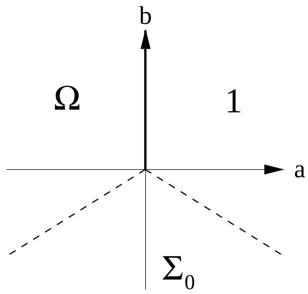

Figure 9: Minima of the potential in eq. 5.2 labeled in eq. 5.3.

5. Lifting states

As mentioned in section 2, the theory spaces we are considering for realistic models have mostly SU(3) sites, one SU(2) × U(1) site, and 3 × 3 matrix link fields transforming as bifundamentals under the SU(3) chiral symmetries. Scalars decompose into triplets, doublets and singlets under the unbroken SU(2). In all the models presented in the previous sections, the triplets, doublets and singlets were classically degenerate. To construct realistic theories we need light doublets, but the triplets and singlets appear as extra adjoint matter that appears to make the doublets into a full SU(3) adjoint multiplets. Finding the minimal 100 GeV field content is an interesting question for phenomenological signatures of the model. The natural question is whether it is possible to remove the light triplets and singlets from the 100 GeV spectrum. Until now we have considered plaquettes that were SU(3) symmetric, and treated triplets, doublets and singlets on equal footing. In this section we generalize plaquette operators to include matrices that are invariant under the SU(2) × U(1) gauge symmetry, but break the SU(3) chiral symmetry. This will allow lifting the extra adjoint matter up to the TeV scale while leaving the doublets at the 100 GeV scale.

The new types of operators that we will consider are of the form:

$$\mathrm{Tr}\ M\Sigma_{\mathbf{0},\mathbf{n}}\Sigma_{\mathbf{n},\mathbf{m}}\cdots \tag{5.1}$$

with $\mathbf{0}$ being the SU(2) × U(1) site and $M = \mathrm{diag}(a, a, b)$. The analysis of the low energy physics of theory spaces that contain these generalized plaquettes proceeds as before, by first gauge fixing and then minimizing the potential plaquette by plaquette. However, the plaquette might not be minimized when the product of link fields is the identity as before. For a plaquette of the form:

$$-\lambda\ \mathrm{Tr}\ M\Sigma + \ \mathrm{h.c.} \tag{5.2}$$

with λ real and positive, there are three different phases for the minimum, depending on the choice of a and b.

$$\begin{aligned} a > 0 \qquad & b > -\frac{1}{2}|a| \qquad \langle\Sigma\rangle = \mathbb{1} \\ a < 0 \qquad & b > -\frac{1}{2}|a| \qquad \langle\Sigma\rangle = \Omega \\ & b < -\frac{1}{2}|a| \qquad \langle\Sigma\rangle = \Sigma_0 \end{aligned} \tag{5.3}$$

with $\Sigma_0 = \exp\left(i\mathbf{T_8}\eta_0\right)$, $\eta_0 = \cos^{-1}(-2b/a)$, and $\Omega =$ diag $(-1 -1, 1)$, $\mathbf{T_8} =$ diag $(1, 1, -2)$. Typically the Σ_0 vacuum is uninteresting because it produces tree level masses for all the fields and we will not consider it any further.

The resulting moduli space might not be SU(3) symmetric, and when the link fields are expanded around the appropriate vacuum, the number of triplet and singlet zero modes might be different than the number of doublet zero modes. To see how this happens, consider a general 3×3 special unitary matrix:

$$Z = \exp(iz) = \exp i \begin{pmatrix} \phi + \eta & h \\ h^\dagger & -2\eta \end{pmatrix} \tag{5.4}$$

then

$$\Omega Z \Omega = \exp(i\Omega z \Omega) = \exp i \begin{pmatrix} \phi + \eta & -h \\ -h^\dagger & -2\eta \end{pmatrix} \tag{5.5}$$

and a relation of the form

$$\Omega Z \Omega Z = \mathbb{1} \qquad \Rightarrow \qquad V = -\lambda f^4 \ \mathrm{Tr}\ Z\Omega Z\Omega \sim \lambda f^2 \ \mathrm{Tr}\ \left(\phi^2 + \eta^2\right) + \cdots \tag{5.6}$$

indicates that around $Z = \mathbb{1}$, the triplet and singlet, ϕ and η are massive while the doublet h is massless. We can now use this tool to lift the triplet and singlet zero modes that were present in the models considered until now. The most obvious set of relations that would produce this result is given by:

$$UVU^{-1}V^{-1} = \mathbb{1} \qquad U\Omega U\Omega = \mathbb{1} \qquad V\Omega V\Omega = \mathbb{1} \tag{5.7}$$

The first relation guarantees the presence of a commutator quartic potential as in the torus, and the last two relations, when expanded around $U, V = \mathbb{1}$ lift the singlet and triplet zero modes.

We now need to build a theory space which yield those relations. We use a very similar procedure to the one described in section 2.1. As before, we first draw the relations using only one site but we now insert Ω as they appear in the relations. We then tile this construction in a way that satisfies the rules mentioned earlier. The insertion of Ω represents a plaquette that is minimized at Ω. Figure 10 shows the building of the theory space in question.

5.1 Minimal model

We can also build simpler theory spaces with the same relations. Consider the two sites model presented in section 2. In addition to the plaquettes in eq. 2.13 we add two new plaquettes containing Ω. The total potential is given by:

$$\begin{aligned} V = & -\lambda_1 X_1 X_2^\dagger X_3 X_4^\dagger - \lambda_2 \ \mathrm{Tr}\ X_1^\dagger X_2 X_3^\dagger X_4 - \\ & - \lambda_3 \ \mathrm{Tr}\ \Omega X_1 X_2^\dagger \Omega X_4 X_3^\dagger - \lambda_4 \ \mathrm{Tr}\ \Omega X_1 X_4^\dagger \Omega X_2 X_3^\dagger \end{aligned} \tag{5.8}$$

The analysis of this model is straight forward. We can gauge fix by setting $X_1 = \mathbb{1}$. We then minimize the first plaquette which gives $X_3 = X_2 X_4$. Minimization of the second

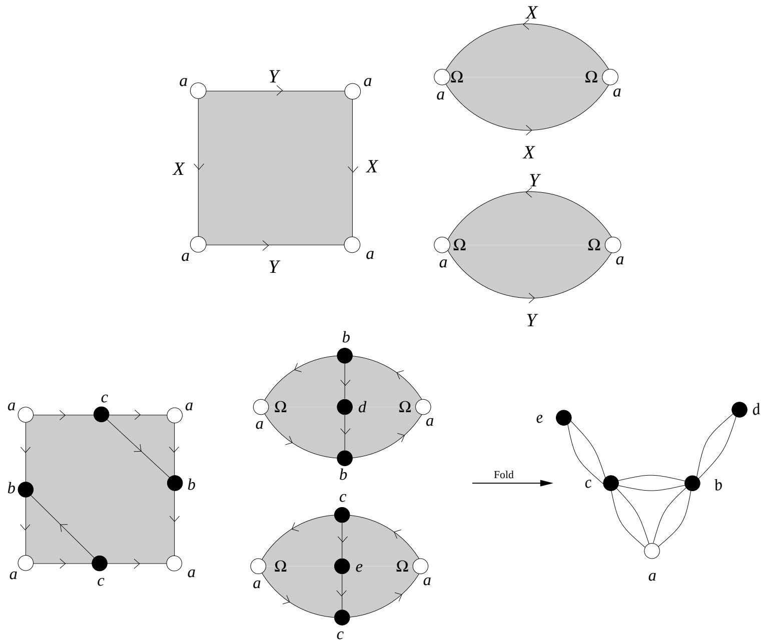

Figure 10: Construction of a theory space with relation in eq. 5.7 that lift triplet and singlet Higgs. The starting point is the first picture where the three relations are drawn with the Ω inserted. The second picture shows a tiling that has no one loop quadratic divergences.

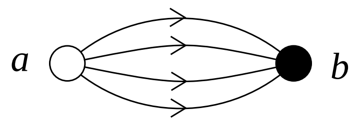

Figure 11: A minimal model for electroweak symmetry breaking by little Higgs.

plaquette gives $X_2 X_4^\dagger X_2^\dagger X_4 = \mathbb{1}$. The third plaquette then requires $\Omega X_2^\dagger \Omega X_2^\dagger = \mathbb{1}$. Finally the fourth plaquette yields $\Omega X_4^\dagger \Omega X_4^\dagger = \mathbb{1}$. Therefore we see that this theory space has the same relations and consequently the same low energy physics as the theory space of figure 10. The spectrum of this theory can also be understood by expanding the plaquettes around the vacuum which we choose to be at $X_i = \mathbb{1}$. Using eq. 5.5, we can see the plaquettes give mass to three combinations of triplets and singlets and to one combination of doublets. One triplet, one singlet and one doublet scalar are eaten by the SU(3) gauge field multiplet that pick up a mass and we are left at low energy with two doublet zero modes. These are the little Higgs of our theory and they pick up a negative mass squared through top Yukawa interaction which can be implemented as in section 4. There is a large stabilizing quartic interaction which is guaranteed by the potential and can be tied to the top quark Yukawa coupling in the manner described in section 4.1. At the TeV scale, the theory contains one doublet, three triplet and three singlet scalars and one multiplet of SU(3) vector bosons. It also contains heavy fermions that were introduced in order to cancel the quadratic divergence associated with the top Yukawa coupling. Because the top Yukawa is in general larger than the gauge couplings and quartic interactions, these heavy fermions will typically be the lightest of the new TeV scale particles.

6. Conclusion

The stability of the weak scale requires new physics at the TeV scale. This physics could be strongly coupled as in technicolor models or weakly coupled as in supersymmetry. There is now a new class of models that stabilize the weak scale with weakly coupled new physics qualitatively different than supersymmetry [1, 17]. Higgs bosons in these theories are pseudo-Goldstone bosons and therefore naturally light. We studied models of this kind that can be described by general theory spaces. This generalize the analysis of [1, 19] which used toroidal theory spaces. The physics however remains the same: the Higgs are pseudo-Goldstone bosons and have their mass protected by approximate chiral symmetries. The quadratic divergences caused by couplings of the Higgs to particles of the low energy theory are softened at the TeV scale by "partners" of the same spin. The theory remains perturbative up to scales of $\sim 10\,\text{TeV}$ where an ultraviolet completion is needed.

The main result of this paper is the development of systematic procedures for extracting the low energy particle content and potential form arbitrary theory spaces and for building theory spaces that produce arbitrary low energy field content and potential. The former consists in calculating the classical moduli space of the theory by first gauge fixing and then minimizing each plaquette, and is equivalent to calculating the fundamental group of the theory space. We thus learn that the low energy physics of a theory space is determined by its topology, and different theory spaces with the same first homotopy group will have the same low energy physics. They will differ only in their TeV scale spectrum.

We also derived two simple properties that a theory space must satisfy in order to be free of quadratic divergences at one loop. This put some mild constraints on the shape of admissible theory spaces. We also showed a simple way of introducing the top Yukawa coupling without reintroducing quadratic divergences. The one loop constraints make for minimal TeV scale physics predictions. To solve the one loop gauge quadratic divergence there must be a W' and B' massive vectors in the 1–2 TeV range. To remove the one loop quadratic divergences from the tree level scalar potential, there must be at least pair of triplets and a pair of singlets in the 100 GeV–1 TeV range and an additional set of triplet, doublet of singlet scalars in the 1–2 TeV range. Finally, for the top quark coupling, a coloured Dirac fermion in the 700 GeV–1 TeV range is necessary. The lack of striking experimental signatures in the 100–500 GeV range is the surprising feature of this class of models. In particular, distinguishing this set of models from supersymmetric models from the two light doublets would be a challenging task at the Tevatron or LHC.

Finally, we made use of the presence of a site of reduced gauge symmetry and introduced generalized plaquettes that are gauge invariant but break the chiral symmetries (the "$\mathbf{T_8}$ plaquette" of [1, 19]). This allowed us to push to the TeV scale the light singlet and triplet scalars that were present before [1, 19, 20] and were the "SU(3) companions" of the Higgs doublets. Using these generalized plaquette we built a minimal model of electroweak symmetry breaking from theory space. It is very similar to the model of [20] but with the light triplet and singlet scalar lifted to the TeV scale. In the 100 GeV region the model has only two Higgs doublets and in the TeV range has three singlet and triplet scalars, one doublet scalar, one SU(3) vector boson multiplet and one coloured fermion.

JHEP08(2002)019

Little Higgs theories are still largely unexplored and there are a lot of model building and phenomenological studies to be done. Interesting possibilities include combining the ideas of [21] with little Higgs, pushing the cutoff to higher energies by using a "cascade" of theory spaces, detailed studies of collider signatures, and cosmological implications.

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